# Homework 1, Math 2406

January 13, 2012

1. Let $p(x), q(x)$ denote the following open statements:

$$p(x) \ : \ x \leq 3 \qquad q(x) \ : \ x + 1 \text{ is odd.}$$

   If the universe consists of all integers, what are the truth values of the following statements:

   a. $q(1)$

   b. $\neg p(3)$

   c. $p(7) \vee q(7)$

   d. $p(3) \wedge q(4)$

   e. $\neg(p(-4) \vee q(-3))$

   f. $\neg p(-4) \wedge \neg q(-3)$.

2. Express the following English statements as logical formulas, by appropriately identifying statments $p(x), q(x), ...$, and then using the symbols $\neg, \wedge, \vee, \exists, \forall, \implies, \iff, \in, \mathbb{Z}$ (and parentheses and other basic mathematical symbols, as needed):

   a. Not all even integers are positive.

   b. For every $\varepsilon > 0$ there is a $\delta > 0$ so that if $|x - 1| < \delta$, then $|x^2 - 1| < \varepsilon$.

   c. All integers have the property that if you add 1 to it, you get back an integer.

   d. A necessary and sufficient condition for $x$ to be an integer is that $x - 1$ is an integer.

3. Construct a truth table for

$$p \;\leftrightarrow\; [(q \wedge r) \rightarrow \neg(s \vee r)].$$

4. Let $p, q$, and $r$ denote statements. Prove or disprove:

$$[p \leftrightarrow (q \leftrightarrow r)] \iff [(p \leftrightarrow q) \leftrightarrow r]$$

5. Prove that if $A \subseteq B$ and $C \subseteq D$, then $A \cap C \subseteq B \cap D$.

6. Using de Morgan's laws for two sets $A$ and $B$, prove the following de Morgan law for three sets $A, B$ and $C$:

$$\overline{A \cup B \cup C} \;=\; \overline{A} \cap \overline{B} \cap \overline{C}.$$