HW 1, Math 2406, Fall 2012

August 24, 2012

1. Prove that
$$[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$$
   is a tautology using truth tables.

2. Using rules of logic mentioned in class (i.e. you don't need to use a truth table), prove that

$$[(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p)] \iff [(p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)]$$

3. Let $p(x), q(x)$ and $r(x)$ be the following statements:

$$
\begin{aligned}
p(x): & \quad x^2 - 7x + 10 = 0 \\
q(x): & \quad x^2 - 2x - 3 = 0 \\
r(x): & \quad x < 0.
\end{aligned}
$$

   a) Determine the truth or falsity of the following statements:
   i) $\forall x[p(x) \rightarrow \neg r(x)]$
   ii) $\forall x[q(x) \rightarrow r(x)]$
   iii) $\exists x[q(x) \rightarrow r(x)]$
   iv) $\exists x[p(x) \rightarrow r(x)]$

   b) Find the answers to part (a) when the universe consists of all positive integers.

   c) Find the canswers to part (a) when the universe contains only the integers 2 and 5.

4. Prove that if n is odd, then $n+13$ is even using proof by contraposition. Then prove it using proof by contradiction.

5. Suppose that A and B are sets. Prove that if $\overline{A} = \overline{B}$, then $A = B$.

6. Suppose that A, B are sets such that $A \cap B = \emptyset$ (this is the "empty set", the set with no elements; when $A \cap B$ is the empty set like this, we say that A and B are *disjoint*). Can it be the case that $\overline{A} \cap \overline{B}$ is NOT the empty set, given that the universe U is non-empty? Justify your answer.