Solution to problem 1

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Solution. To solve this problem we first must find an orthogonal basis for the space spanned by x and $x+1$ (because, recall, projection is defined in terms of an orthogonal basis): we can start by letting $b_1 = x$ and then to find b_2 we subtract the projection of $x+1$ onto the space spanned by x; so,

$$b_2 \ = \ x+1-x\frac{\int_0^1 x^2(x+1)dx}{\int_0^1 x^3 dx} \ = \ x+1-\frac{x(1/4+1/3)}{1/4} \ = \ x+1-7x/3 = -4x/3+1.$$

You can get rid of denominators and negatives here – it's perfectly legal (b_1 and b_2 will still be orthogonal) – just multiply by -3, and you get $b_2 = 4x-3$.

And now, our projection is just:

$$\begin{aligned} p(x^2+x+1) &= \frac{< x^2+x+1, x >}{< x, x >}x + \frac{< x^2+x+1, 4x-3 >}{< 4x-3, 4x-3 >}(4x-3) \\ &= \frac{(1/5+1/4+1/3)}{(1/4)}x + \frac{4/5+1/4+1/3-3/2}{4-8+9/2}(4x-3) \\ &= \frac{47x}{15} + \frac{48+15+20-90}{240-480+270}(4x-3) \\ &= \frac{47x}{15} + \frac{-7(4x-3)}{30} \\ &= 11x/5 + 7/10. \end{aligned}$$