Solution to problem 2

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Solution. I will assume you know how to prove it's a subspace.

Ok, so you have polynomials of degree at most 5 satisfying $f(x) - 2f(x+1) + f(x+2) = 0$. Let's see where $f(x) \to f(x) - 2f(x+1) + f(x+2)$ maps the term x^a, where $a = 0, 1, 2, 3, 4$, or 5: we get that 1 and x get sent to 0; but x^2 gets sent to $x^2 - 2(x+1)^2 + (x+2)^2 = 2$, which is NOT 0; x^3 gets sent to $x^3 - 2(x+1)^3 + (x+2)^3 = 6x + 6$; x^4 gets sent to a degree 2 polynomial; and x^5 gets sent to a degree 3 polynomimal.

So, if $f(x)$ is a polynoial of degree d, writing it as a linear combination of $1, x, x^2, x^3, x^4$ and x^5, it's clear that $f(x) - 2f(x+1) + f(x+2) = 0$ if and only if f has degree at most 1. Therefore, the space is 2 dimensional.