

Study Sheet for Math 2406 Midterm 1, Fall 2012

September 19, 2012

1. Know the basics of logic, and know what the symbols

$$\vee \wedge \neg \implies \longrightarrow \forall \exists \leftrightarrow \iff$$

mean. Know what a truth table is; know how to construct one; and know how to use it to prove a logical proposition. Know what “proof by contradiction” is; know what “proof by contraposition” is. Know that when you negate a statement, you change quantifiers around. Know de Morgan’s laws.

2. Know what is meant by “type I” and “type II” induction. Know how to prove basic statements using induction, such as to show that $1 + 2 + \cdots + n = n(n+1)/2$. Also, study the proof that every integer $n \geq 2$ can be written as a product of prime numbers (using type II induction). Recall the proof on the number of moves needed to solve the Towers of Hanoi puzzle. Know how to use induction to prove inequalities.
3. Know basic facts about sets, and in particular what the symbols $\cup \cap \emptyset \in \subseteq \subset \supseteq \supset$ and \overline{A} all mean. Know what is meant by the “universal set” or “universe”. Know what injection, surjection, and bijection all mean; know that $|A| = 5$ means that there exists a bijection $\varphi : A \rightarrow \{1, 2, \dots, 5\}$; and know that $|A| = |B|$ means there exists a bijection $\psi : A \rightarrow B$. Know what is meant by “countable” and “uncountable”. Know that the real numbers \mathbb{R} form an uncountable set; know that $\mathbb{Z}, \mathbb{Q}, \mathbb{N}$ are all countable. Know the de Morgan laws of sets, and be able to prove them. Know that the statement “ $A \subseteq B$ ” can be rendered in logical symbols as “ $\forall x(x \in A \implies x \in B)$ ”. Know the de Morgan laws.

4. Know what is meant by an equivalence relation; and know what the terms symmetric, reflexive, and transitive mean. Know how to prove that certain relations are equivalence relations – see the note on the course webpage about equivalence relations.
5. Know basic properties of the vector space \mathbb{R}^n . Know how to think about vectors as points in the space; know how to draw vectors; know how to add vectors; know what is meant by parallel vectors; know what is meant by “a scalar multiple of a vector”. Know what is meant by the dot product $A \cdot B$, and know its basic properties; know the difference between the complex dot product and the real dot product; know what is meant by $\|A\|$, and how to express it in terms of dot product as $(A \cdot A)^{1/2}$; know the interpretation of the dot product in terms of projections; know what is meant by “orthogonal vectors”. Know the Cauchy-Schwarz inequality for \mathbb{R}^n and \mathbb{C}^n . Know the triangle inequality $\|A + B\| \leq \|A\| + \|B\|$.
6. Know what is meant by “the linear span of the vectors x_1, \dots, x_k ”, which, in symbols, is written $L(\{x_1, \dots, x_k\})$ (not to be confused with $L(P; A)$, our notation for a line). Know what it means for a set of vectors in \mathbb{R}^n to be “linearly dependent” (often written as just ‘dependent’) and “linearly independent” (often written as just ‘independent’). Know the fact that if $x_1, \dots, x_{k+1} \in L(\{y_1, \dots, y_k\})$, then x_1, \dots, x_{k+1} must be linearly dependent. Know that if x_1, \dots, x_k are “orthogonal” (meaning that the dot-products $x_i \cdot x_j = 0$ for all $i \neq j$; and that no $x_i = 0$), then they are linearly independent; and know how to prove this. Know the terms basis and dimension. Know basic properties, such as that any n linearly independent vectors in \mathbb{R}^n necessarily form a basis; and that any two bases have the same number of elements (that number is the dimension).