

Homework 5, Math 3012, Fall 2009

October 20, 2009

1. Suppose that $A_1, A_2 \subseteq A$ and $B_1, B_2 \subseteq B$ are all finite sets. Show that

$$\begin{aligned} & |(A_1 \times B_1) \cup (A_1 \times B_2) \cup (A_2 \times B_1) \cup (A_2 \times B_2)| \\ &= (|A_1| + |A_2| - |A_1 \cap A_2|)(|B_1| + |B_2| - |B_1 \cap B_2|). \end{aligned}$$

2. Suppose that $f : A \rightarrow B$ is an injective function from the finite set A to the finite set B . If $|A| = |B|$, show that f is also surjective.

3. Suppose that A is a set of 10 elements. How many relations $A \rightarrow A$ are *symmetric* (which means that (a, b) is part of the relation if and only if (b, a) is part of the relation)?

4. Let

$$\Delta := 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19.$$

How many quadruples

$$(x_1, x_2, x_3, x_4) \in \mathbb{Z}_{\geq 2} \times \mathbb{Z}_{\geq 2} \times \mathbb{Z}_{\geq 2} \times \mathbb{Z}_{\geq 2}$$

are there that satisfy

$$x_1 x_2 x_3 x_4 = \Delta ?$$