Homework 5, part 2, Math 3012, Fall 2009

October 26, 2009

1. Suppose that A, B, and C are finite sets such that there exists a surjection $\varphi : A \to C$ and an injection $\psi : B \to C$, then there exists a surjection $\theta : A \to B$.

2. Suppose that $\varphi : A \to A$ is a bijection. Show that there exists an integer $n \geq 1$ such that if you compose φ with itself n times then it equals the identity mapping. That is, show that there exists $n \geq 1$ such that for all $a \in A$,

$$\varphi^n(a) \ = \ (\varphi \circ \cdots \circ \varphi)(a) \ = \ a.$$

(Hint: Note that any bijection φ is invertible. Form the sequence of mappings $\varphi, \varphi^2, \varphi^3, ...$ Since there are only a finite number of bijections from $A \to A$, at some point this list must cycle – i.e. you get $\varphi^m = \varphi^n$ for some pair m, n satisfying $m < n$. Now try to think about how to use this.)