Combinatorics Midterm 1

September 17, 2009

1.

a. Find all possible permutations of the 11 letters in the word ABRACADABRA. (You can leave your answer in terms of factorials.)

b. How solutions are there to

$$42 = x_1 + x_2 + x_3 + x_4,$$

where x_1, x_2, x_3 and x_4 are integers such that $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$, and $x_4 \geq 4$?

2. Determine the number of sequences of 5 integers chosen from the integers 1,2,3,...,20, such that none of these 5 integers are consecutive (for example, you would not include the 5 numbers 1,3,4,10,12 in your count, because 3 and 4 are consecutive). Hint: One way to solve this is the method of barriers.

3. Using the fact that for integers $n \geq 1$,

$$\left(1 + \frac{1}{n}\right)^n < e \approx 2.718281828...,$$

use mathematical induction to prove that

$$n! > \left(\frac{n}{e}\right)^n.$$

for all integers $n \geq 1$. (One can also prove this using calculus by integrating $\ln(x)$.)

4. Using the Euclidean algorithm, find integers x and y such that

$$97x + 71y = 1.$$

5. Suppose that $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7, 8\}$.

a. Determine $A \cap B$.

b. Determine $A \setminus B$.

c. Suppose that the unversal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Determine $\overline{A}$ and $\overline{B}$.

d. Suppose that C and D are subsets of some universal U. If $C \subseteq D$, then show that $\overline{D} \subseteq \overline{C}$.