Practice problems for Midterm 2, Math 3215, Summer 2009

July 10, 2009

1.

a. Determine the moment generating function of the exponential random variable X whose probability density function (pdf) is $f(x) = 3e^{-3x}$.

b. Using your answer from part a, determine the sixth moment of X; that is, determine $\mathbb{E}(X^6)$.

2. Suppose that (X, Y) is a two-dimensional random variable having probability density function

$$f(x, y) \;=\; \frac{1}{2\pi} e^{-(x^2+y^2)/2}.$$

a. Find the marginal probability density function $g(x)$ for the random variable X.

b. What is the expectation of the random variable $Z = X + Y$?

c. What is the expectation of $W = XY$?

3. Suppose that the number of particles that a radioactive source emits in a given one minute time window has a Poisson distribution with parameter $\lambda = 10$. Determine the variance of the distribution of the number of particles emitted in three minute time windows.

4. Recall that X is a chi-squared random variable with n degrees of freedom if its pdf is given by

$$f(x) \;=\; \begin{cases} \frac{x^{n/2-1}e^{-x/2}}{2^{n/2}\Gamma(n/2)}, & \text{if } x \geq 0; \\ 0, & \text{if } x < 0, \end{cases},$$

where, recall, the gamma function is given by

$$\Gamma(x) \;=\; \int_0^\infty e^{-t}t^{x-1}dt.$$

Now suppose that X has n degrees of freedom, where n is even, while Y has $2n$ degrees of freedom. Assume $n \geq 2$. Which of the following is more probable (defend your answer)?

$$\mathbb{P}(X > 2n) \;\text{ or }\; \mathbb{P}(Y > 2n)$$

5. Can you think of an example of where $\mathbb{E}(X+Y) \neq \mathbb{E}(X) + \mathbb{E}(Y)$?