

Study Sheet for Math 3225, Final Exam

December 13, 2010

This test will NOT be open note; however, I will give you some selected notes from the course to use during the final.

1. Know basic definitions and results from set theory; for example, know the two forms of de Morgan's law, know distributive rule of intersection and union (which says $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$). Also know injective, surjective, bijective maps, and how to prove that various maps have these properties.

2. Know the definition of a sigma algebra: S is a sigma-algebra means that

i) S contains the empty set.

ii) If $A \in S$, then $\bar{A} \in S$

iii) If A_1, A_2, \dots is a countable collection of sets in S , then their union belongs to S .

3. Know the Kolmogorov axioms of probability: $P : \sigma \rightarrow [0, 1]$ is a probability measure from the sigma-algebra σ consisting of certain subsets of S (the sample space) if

i) $P(S) = 1$

ii) If A_1, A_2, \dots is a disjoint collection of sets in σ , then $P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$.

4. Know the monotone convergence theorem for sets (there is another version of the MCT, which is more difficult). Know how to use it to, for example, calculate the probability of certain sets that are made up of infinitely many intervals.

5. Know how to prove basic probability inequalities and identities, such as $P(A \cup B) \leq P(A) + P(B)$, and inclusion-exclusion.

6. Know the definition of independent events. Know how to prove various consequences of independence, such as: A, B independent implies $P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$.

7. Know the definition of a random variable, and independent random variables.
8. Know some basic random variables, such as
 - a) Bernoulli
 - b) Poisson
 - c) Binomial
 - d) Geometric
 - e) Exponential
 - f) Normal
 - g) Gamma
 - h) Chi-square
9. Know Bayes's Theorem, and applications.
10. Know how to compute expectation and variance, and know the tower property of expectation, and its applications.
11. Know Markov's inequality and Chebychev's inequality.
12. Know the statement of the Central Limit Theorem, as well as the (weak) law of large numbers. Also, know the proof of the LLN.
13. Know how to apply the central limit theorem to approximate sums of independent random variables.
14. Know how to apply the central limit theorem to find confidence intervals for problems where the variance is known (see the statistical sampling notes, the pedagogical example). And know how to find them when you have a sample from a normal distribution, but don't know the mean or the variance.
15. Know the meaning of "null hypothesis", "alternate hypothesis", Type I error, Type II error. Know how to compute the probability of making Type I and Type II errors. Know how to set of the rejection region, and know how to choose the test statistic. Know how to find the p -value for a given test.
16. Know how to compute moment generating functions for basic random variables, such as uniform, Poisson, Chi-square, Gamma, and exponential.
17. Know how to use moment generating functions to prove things, such as that sums of certain independent random variables have a certain distribution.
18. Know how to use moment generating functions to compute high moments of random variables.
19. Know how to compute the pdf for a function of several random variables. E.g. suppose X and Y are independent and normal with mean 0 and variance 1, determine the pdf of $Z = |XY|$.

20. Know properties of conditional expectation, such as the tower property, and know how to apply it to problems (recall the radioactive source problem).
21. Know how to do a Chi-square test (I will give you chi-square tables for any such problems).
22. Know how to use the chi-square distribution to determine goodness of fit of curves to data (see notes on chi-square).
23. Know how to find a maximum likelihood estimate for a parameter, given that a distribution is of a certain type (such as normal).
24. Know basic properties of the gamma function, and be able to prove things about it.
25. Know what a Markov process is, and its matrix representation. Also, know various properties of the matrix, such as the sizes of its eigenvalues.
26. Know how to determine stationary distributions.
27. Know how to compute the probability that the MP is in state j , at time n , given that it starts in state i .