Study Sheet for Math 3225, Exam 1, Fall 2010

October 8, 2010

1. Know basic definitions and results from set theory; for example, know the two forms of de Morgan's law, know distributive rule of intersection and union (whichs says $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$). Also know injective, surjective, bijective maps, and how to prove that various maps have these properties.

2. Know the definition of a sigma algebra: S is a sigma-algebra means that

   i) S contains the empty set.

   ii) If $A \in S$, then $\overline{A} \in S$

   iii) If $A_1, A_2, ...$ is a countable collection of sets in S, then their union blongs to S.

3. Know the Kolmogorov axioms of probability: $P : \sigma \to [0, 1]$ is a probability measure from the sigma-algebra σ consisting of certain subsets of S (the sample space) if

   i) $P(S) = 1$

   ii) If $A_1, A_2, ...$ is a disjoint collection of sets in σ, then $P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$.

4. Know the "union bound" $P(A_1 \cup A_2 \cup \cdots) \leq \sum_i P(A_i)$, even if the A_i's are not disjoint.

5. Know the monotone convergence theorem for sets (there is another version of the MCT, which is more difficult). Know how to use it to, for example, calculate the probability of certain sets that are made up of infinitely many intervals.

6. Know how to prove basic probability inequalities and identities, such as $P(A \cup B) \leq P(A) + P(B)$, and inclusion-exclusion.

7. Know the definition of independent events. Know how to prove vaious consequences of independence, such as: A, B independent implies $P(\overline{A} \cap \overline{B}) = P(\overline{A})P(\overline{B})$.

8. Know the definition of a random variable, and independent random variables.

9. Know the difference between "disjoint events" and "independent events".

10. Know the definition of probability density functions (pdf's for short), and know the definition of a cumulative distribution function.

11. Know some basic random variables, such as

    a) Bernoulli

    b) Poisson (and know Poisson Processes)

    c) Binomial

    d) Geometric

    e) Exponential

    f) Normal

12. Know Bayes's Theorem, and applications.

13. Know how to compute expectation and variance of a r.v.

14. Know Markov's inequality and Chebychev's inequality.

15. Know how to compute the pdf for a random variable Y that is a function of some other random variable X. The idea is to pass to cdf's by finding $P(Y \leq t)$, and then take a derivative.