Homework 2, Math 3225, Fall 2010

October 7, 2010

1. Suppose that X and Y are independent normal random variables with mean μ and variance σ^2. Show that $\lambda X + \mu Y$ is also a normal random variable for arbitrary constants λ and μ, not both 0. Determine its mean and variance.

2. The number of people arriving at a fast food drive through in any given 2 minute interval obeys a Poisson process with mean 1. Suppose that the waiters can only process 3 orders in any given 4 minute interval. What is the expected number of people that leave the drive through with their orders filled in any given 4 minute time interval. Recall that X is a Poisson random variable with mean λ means that $P(X = j) = e^{-\lambda}\lambda^j/j!$.

3. Fermat's Last Theorem in number theory, proved by Andrew Wiles (and so should be named Wiles's Theorem), asserts that the equation

   $$x^n + y^n \;=\; z^n$$

   has no solutions with $x, y, z \geq 1$ all integers when $n \geq 3$.

   Using this fact, show that the following problem cannot be solved: Two urns contain the same total number of balls, and some balls are white and some are black. From each of these urns, $n \geq 3$ balls are drawn with replacement. Find the number n and composition of each urn if the probability all the balls taken from the first urn are white is equal to the probability that all the balls drawn from the second urn are either all white or all black.

4. Prove that if X is a Poisson random variable with parameter λ, then $V(X) = \lambda$.

5. Suppose you have a collection of days. 10 percent are rainy days, and 90 percent are dry days. If you select one of the rainy days at random, there is a 20 percent chance it will be a spring day; and if you select a dry day at random, there is an 80 percent chance it *won't* be a spring day. If you select a spring day at random, what is the probability it was a rainy day?