

Math 4107, exam 1

September 22, 2009

Each question is worth 20 points.

1. Define the following terms.
 - a. Normal subgroup.
 - b. Homomorphism.
 - c. Innerautomorphism.
2. List all the elements of order 2 from the group S_4 .
3. Find integers k and ℓ , satisfying

$$55k + 21\ell = 1, \quad |k| \leq 20, \quad |\ell| \leq 54.$$

Don't just write down the answer. Work it using the Euclidean algorithm (e.g. Knuth's algorithm), so that I know you know some good mathematics.

4. Consider the set of all 2×2 matrices of the form

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}.$$

Prove that the set of all such matrices forms a group under matrix multiplication.

5.
 - a. First, show that D_4 has a normal subgroup of order 2 (in fact, it is a quite special type of normal subgroup...). Note that D_n for all n even has such a subgroup, and for n odd there is no such subgroup.
 - b. Show that the group of automorphisms of the group D_4 , denoted by $\text{Aut}(D_4)$, contains at least 4 elements.