Midterm 1, Math 4107, Fall 2009

September 24, 2009

1. Define the following terms.
   a. Normal subgroup.
   b. Group.
   c. Homomorphism.
   d. First Isomorphism Theorem.
   e. Fundamental Theorem of Arithmetic.

2. Suppose that G is a group of order

$$105 \;=\; 3 \cdot 5 \cdot 7,$$

and suppose that G has a subgroup of order 15 and a subgroup of order 35. Prove that G contains an element of order 5.

3. Show that the dihedral group D_6 contains a subgroup isomorphic to S_3 (the symmetric group on 3 symbols).

4. Find integers x and y satisfying

$$322x + 189y \;=\; \gcd(322, 189),\ |x| \leq 188,\ |y| \leq 321.$$

Show your work (if you just write down the answer, you will not get credit – the point of this problem is to demonstrate to me that you know how to find x and y efficiently).

5. Determine the number of abelian subgroups of S_4 having order 4, and list them out.