Midterm 1, Math 4108, Spring 2010

March 1, 2010

1. Define the following terms.
  a. Algebraic extension.
  b. Unique Factorization Domain.
  c. Splitting field.
  d. Eisenstein's criterion.
  e. algebraic number.

2. Compute the degree of the extension $F[x]/(x^2 - 2)$ over $\mathbb{Q}$, where $F = \mathbb{Q}(i)$, where $i^2 = -1$. Explain your answer.

3. Determine the gcd of the polynomials $f(x) = x^5 + 3x^3 + 2x^2 + 2x + 2$ and $g(x) = x^5 + x^4 + 3x^3 + 4x^2 + 4x + 2$, where we think of $f, g \in \mathbb{F}_7[x]$.

4. Suppose that K is an algebraic extension of a characteristic 0 field F, in which every $\alpha \in K$ satisfies some polynomial of degree at most $B > 0$ (i.e. we have an upper bound on the degree of elements in K). Prove that K is in fact a finite extension of F. (Hint: Primitive element theorem. If you don't know what it says, ask me in class [it is was not on the study sheet... and this problem is too nice not to be on an exam].)

5. Suppose that $f(x) \in F[x]$ is a degree n irreducible polynomial, and that E is some finite extension of F in which $f(x)$ has a root. Prove that n divides $[E : F]$.