Math 4108 midterm 2, Spring 2010

April 30, 2010

1. Define the following terms

    a. Galois group of a field K over a field F (Also, what properties must K have in order for the Galois group to even exist?)

    b. Lower Central Series.

    c. Fundamental Theorem of Finitely Generated Modules over a PID.

    d. normal field extension.

    e. nilpotent group.

2. Compute the derived series for S_4. Explain your work.

3. Compute the Galois group for the polynomial $x^4 - 2$. What basic group is it isomorphic to?

4. Suppose that $\alpha, \beta \in \mathbb{F}_{p^n}$. Let H_α and H_β be the multiplicative subgroups generated by the powers of α and β, respectively. Prove that

$$|H_\alpha| \;=\; |H_\beta| \quad \Longrightarrow \quad H_\alpha = H_\beta.$$

5. Prove that the length of the Upper Central Series for a nilpotent group is always at least as long as the Derived Series for the group (Use whatever facts about Upper and Lower series you need – if you are unsure of what you are allowed to use, just ask me.)