

# Homework on Derived Series, and Upper and Lower Central Series

April 12, 2010

1. We say that a subgroup  $H$  of a group  $G$  is *characteristic* if every automorphism  $\sigma : G \rightarrow G$  maps  $H \rightarrow H$ . Note that if  $G$  has only a single subgroup  $H$  of some order  $n$ , then  $H$  is characteristic.

Show that the elements of the upper central series are all characteristic subgroups of  $G$ .

2. Assume  $G$  is a finite group. Using the characterization of nilpotent from class, prove that  $G$  is nilpotent if and only if  $G$  has a subgroup of each order dividing  $|G|$ .
3. Find the upper and lower central series for  $A_4$  and  $S_4$ .