Combinatorial and Analytic Methods in Number Theory

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Text: I will use several texts, and so there will not be a specific one required. I will hand out lecture notes that I write, as well as photocopy notes from various texts relevant to the lecture at hand.

Summary: The goal of this course will be to introduce various tools from combinatorial and analytic number theory that are useful for solving diophantine problems (rational solutions to polynomial equations), as well as estimating various arithmetical functions (such as the prime number counting function $\pi(x)$). Many of these tools will be fairly elementary, but surprisingly require a lot of effort to properly master.

Here is a partial list of tools that I would like to cover, though it is not clear how many I will actually be able to get to:

- Gauss and Jacobi sums, and the estimation of the number of points on special varieties in finite fields.
- Sum-Product inequalities of Bourgain, Katz, and Tao, with applications to diophantine problems.
- An introduction to the sieve methods of Brun, Selberg, and Montgomery and Vaughan, with applications.
- Elementary techniques for the estimation of sums of arithmetical functions.
- Contour methods and estimation of sums of arithmetical functions, with applications to Antal Balog's smooth number theorem.

• The Mellin Transform and Meinardus's theorem, with applications; specifically, giving a quick proof of the asymptotic formula for the partition function $p(n)$.

• The prime number theorem (PNT), PNT for arithmetic progressions and the class number equation.

• Roth's proof on sets with three-term arithmetic progressions.

• Equidistribution and Fourier analysis: Weyl sums.

• Basic Ergodic theory: Proof and applications of the multiple Birkhoff ergodic theorem.

• Pade approximations, with applications to diophantine equations.